1980 Q1 H.P.

1. A boat has to travel by the shortest route to the point $4{\cdot}25\vec{j}$ km and then return immediately to its starting point at the origin. The velocity of the water is $(8\sqrt{2}\,\vec{i} - 8\sqrt{2}\,\vec{j})$ km/hour and the boat has a speed of 18 km/hour in still water.

If $a\vec{i} + b\vec{j}$ is the velocity of the boat on the outward journey, find a and b and the time taken for the outward journey, leaving your answer in surd form. Find, also, the time taken for the whole journey.

$\vec{V}_{WG} = 8\sqrt{2}\,\vec{i} - 8\sqrt{2}\,\vec{j}$: $\vec{V}_{BW} = a\vec{i} + b\vec{j}$: $a^2 + b^2 = 18^2$

$\vec{V}_{BG} = \vec{V}_{BW} + \vec{V}_{WG} = (a + 8\sqrt{2})\,\vec{i} + (b - 8\sqrt{2})\,\vec{j}$

As $\vec{V}_{BG}$ in $\vec{j}$ direction $\Rightarrow a = -8\sqrt{2}$

$\Rightarrow a^2 + b^2 = 128 + b^2 = 18^2$

$\Rightarrow b^2 = 196 = 14^2$

On outward journey $\vec{V}_{BG} = (14 - 8\sqrt{2})\,\vec{j}$ Km/hr

Distance Travelled $= 4\frac{1}{4} = \frac{17}{4}$ Km

Time taken: $T_O = \dfrac{17}{4(14 - 8\sqrt{2})}$

On Return Journey $\vec{V}_{BG} = (-14 - 8\sqrt{2})\,\vec{j} = -(14 + 8\sqrt{2})\,\vec{j}$

Time taken $T_R = \dfrac{17}{4(14 + 8\sqrt{2})}$

Total time $T = \dfrac{17}{4}\left(\dfrac{1}{14 - 8\sqrt{2}} + \dfrac{1}{14 + 8\sqrt{2}}\right)$

$= \dfrac{17}{4}\left(\dfrac{28}{14^2 - (8\sqrt{2})^2}\right)$

$= \dfrac{17}{4} \times \dfrac{28}{68}$

$\Rightarrow T = \dfrac{7}{4}$ hrs